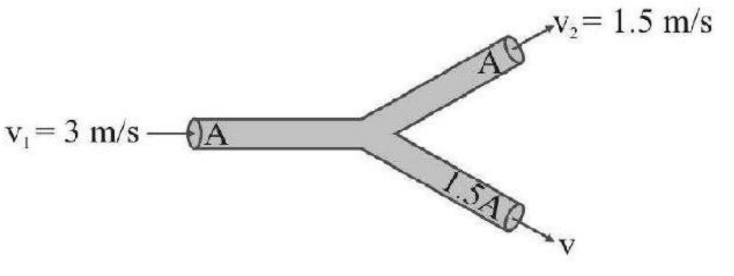
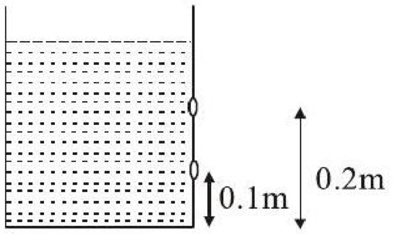
Mechanical Properties of Fluids

1. The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth (in metre) of the centre of the opening from the level of water in the tank is
2. Water flows into a large tank with flat bottom at the rate of 10^{-4} m^3 s^{-1}. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height (in cm) is:
3. A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides, in cm, will be :
4. Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is : (density of water $= 1000 \mathrm{~kg/m^3}$, coefficient of viscosity of water $= 1 \mathrm{mPa} s$)
5. An incompressible liquid flows through a horizontal tube shown in the following fig. Then the velocity v (in $\mathrm{m/s}$) of the fluid is



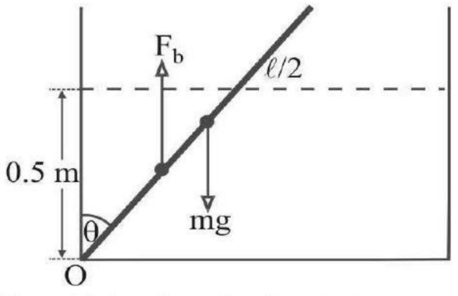
6. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum mass (kg) that can be put on the block without fully submerging it under water? [Take, density of water $= 10^3$ $\mathrm{kg/m^3}$]
7. A submarine experiences a pressure of 5.05×10^6 Pa at depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. Then $d_2 - d_1$ (in metre) is approximately (density of water $= 10^3 \mathrm{~kg/m^3}$ and acceleration due to gravity $= 10 \mathrm{~ms^{-2}}$):
8. Water from a tap emerges vertically downwards with an initial speed of $1.0 \mathrm{~ms^{-1}}$. The cross-sectional area of the tap is $10^{-4} \mathrm{~m^2}$. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area (in m^2) of the stream, 0.15 m below the tap would be : [Take g $=$ $10 \mathrm{~ms^{-2}}$]
9. A solid sphere, of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity h. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2, when falling through the same fluid, the ratio (v_1/v_2) equals $\frac{9}{x}$. Find the value of x.
10. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is:
11. A long cylindrical drum is filled with water. Two small holes are made on the side of the drum as shown in the figure. The depth (in m) of the liquid in the drum if the ranges of water from the holes are equal.



12. A glass U-tube is such that the diameter of one limb is 3.0 mm and that of the other is 6.00 mm. The tube is inverted vertically with the open ends below the surface of water in a beaker. What is the difference between the heights (in m) to which water rises in the two limbs? Surface tension of water is $0.07\ \mathrm{N/m}$. Assume that the angle of contact between water and glass is $0°$.

13. Two pistons of hydraulic press have diameters of 30.0 cm and 2.5 cm. What is the force (in kg-wt) exerted by larger piston, when 50.0 kg -wt is placed on the smaller piston?

14. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown in Figure. The tank is filled with water up to a height of 0.5 m. The specific gravity of the plank is 0.5. Find the angle (in degree) that the plank makes with the vertical in the equilibrium position (Exclude the case $\theta = 0$).



15. A cube of ice of edge 4 cm is placed in an empty cylindrical glass of inner diameter 6 cm. Assume that the ice melts uniformly from each side so that it always retains its cubical shape. Remembering that ice is lighter than water, find the length (in cm) of the edge of the ice cube at the instant it just leaves the contact with the bottom of the glass.

SOLUTIONS

1. **(4.8)** Here, volume tric flow rate

$$=\frac{0.74}{60}=\pi r^2 v=\left(\pi\times4\times10^{-4}\right)\times\sqrt{2gh}$$

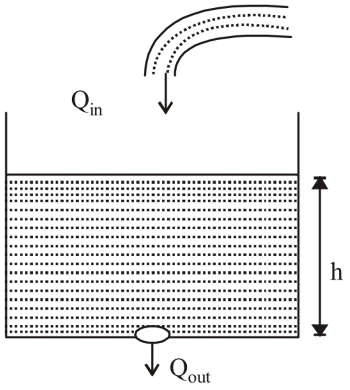
$$\Rightarrow\sqrt{2gh}=\frac{74\times100}{240\pi}\Rightarrow\sqrt{2gh}=\frac{740}{24\pi}$$

$$\Rightarrow 2gh=\frac{740\times740}{24\times24\times10}\left(\because\pi^2=10\right)$$

$$\Rightarrow h=\frac{74\times74}{2\times24\times24}\approx4.8\,m$$

i.e., The depth of the centre of the opening from the level of water in the tank is 4.8 m

2. **(5)**



Since height of water column is constant therefore, water inflow rate (Q_{in})
= water outflow rate

$Q_{in} = 10^{-4}\ m^3s^{-1}$

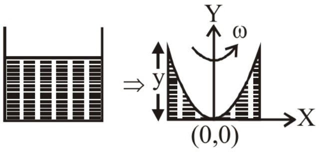
$Q_{out} = Au = 10^{-4}\times\sqrt{2gh}$

$\therefore\ 10^{-4} = 10^{-4}\times\sqrt{20\times h}$

$\therefore\ h = \frac{1}{20}\ m = 5\ cm$

3. **(2)** Using $v^2 = u^2 + 2gy$ $[\therefore u=0 \text{ at } (0,0)]$

$v^2 = 2gy$ $[\therefore v=\omega x]$



$$\Rightarrow y=\frac{\omega^2x^2}{2g}=\frac{(2\times2\pi)^2\times(0.05)^2}{20}\simeq2cm$$

4. **(2 × 10⁴)** Rate of flow of water (V) = 100 lit/min

$$=\frac{100\times10^{-3}}{60}\times\frac{5}{3}\times10^{-3}\text{m}^3$$

∴ Velocity of flow of water (v)

$$=\frac{V}{A}=\frac{5\times10^{-3}}{3\times\times(5\times10^{-2})^2}$$

$$=\frac{10}{15\pi}=\frac{2}{3\pi}\text{ m/s }=0.2\text{ m/s}$$

∴ Reynold number (N_R) $=\dfrac{Dv\rho}{\eta}$

$$=\frac{(10\times10^{-2})\times\frac{2}{3\pi}\times1000}{1}=2\times10^4$$

5. **(1)**

$$A\,v = A_1v_1 + A_2v_2$$

or $A\times3 = A\times1.5+1.5A\times v$

∴ $v = 1$ m/s

6. **(87.5)** When a body floats then the weight of the body = upthrust

$$\therefore (50)^3\times\frac{30}{100}\times(1)\times g=M_{\text{cube}}g \quad \text{...(i)}$$

Let m mass should be placed, then

$$(50)^3\times(1)\times g=(M_{\text{cube}}+m)g \quad \text{...(ii)}$$

Subtracting equation (i) from equation (ii), we get

⇒ $mg = (50)^3 \times g\,(1 - 0.3) = 125 \times 0.7 \times 10^3$ g

⇒ m = 87.5 kg

7. **(300)** $P_1 = P_0 + \rho g d_1$

$P_2 = P_0 + \rho g d_2$

$\Delta P = P_2 - P_1 = \rho g \Delta d$

$3.03 \times 10^6 = 10^3 \times 10 \times \Delta d$

⇒ Dd ≃ 300 m

8. **(5 × 10⁻⁵)** Using Bernoullie's equation

$$P+\frac{1}{2}(v_1^2-v_2^2)+\rho gh=P$$

$$\Rightarrow v_2^2=v_1^2+2gh$$

$$\Rightarrow v_2=\sqrt{v_1^2+2gh}$$

Equation of continuity

$A_1v_1 = A_2v_2$

$$(1\text{ cm}^2)\ (1\text{ m/s}) = (A_2)\left(\sqrt{(1)^2+2\times10\times\frac{15}{100}}\right)$$

$10^{-4} \times 1 = A_2 \times 2$

$$\therefore\quad A_2=\frac{10^{-4}}{2}=5\times10^{-5}\text{ m}^2$$

9. **(1)** $27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r^3$

or $r = \frac{R}{3}$.

Terminal velocity, $v \propto r^2$

$\therefore \quad \frac{v_1}{v_2} = \frac{r_1^2}{r_2^2}$

or $v_2 = \left(\frac{r_2}{r_1}\right)^2 v_1 = \left(\frac{R/3}{R}\right)^2 v_1 = \frac{1}{9}$

or $\frac{v_1}{v_2} = 9$.

10. **(0.6)** $Mg = \left(\frac{4V}{5}\right)\rho\omega g$

or $\left(\frac{M}{V}\right) = \frac{4\rho\omega}{5}$

or $\rho = \frac{4\rho\omega}{5}$

When block floats fully in water and oil, then

$Mg = F_{b_1} + F_{b_2}$

$(\rho V)g = \left(\frac{V}{2}\right)\rho_{\text{oil}}\, g + \frac{V}{2}\rho\omega g$

or $r_{\text{oil}} = \frac{3}{5}\rho\omega = 0.6\rho\omega$

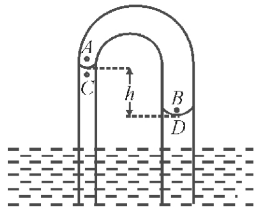
11. **(0.3)** The range is equal for the hole positions h and $(H - h)$ from free surface (H is the height of liquid). So depth of the liquid in the drum will be 0.3 m.

12. **(4.76×10^{-3})** Let P_A and P_B are the pressures at points A and B respectively. The pressure at point C,

$$P_C = P_A - \frac{2T}{R_1}$$

where $\quad R_1 = \frac{r_1}{\cos 0°} = r_1$

The pressure at point D, $P_D = P_B - \frac{2T}{R_2}$



where, $\quad R_2 = \frac{r_2}{\cos 0°} = r_2$

If h is the difference between heights rise in two limbs, then

$$P_D - P_C = h\rho g$$

or $\left(P_B - \frac{2T}{R_2}\right) - \left(P_A - \frac{2T}{R_1}\right) = h\rho g$

As $P_A = P_B$ and $R_1 = r_1 = 1.5$ mm, $R_2 = r_2 = 3.0$ mm, so

$$2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = h\rho g$$

$$0.2 \times 0.07\left(\frac{1}{1.5\times10^{-3}} - \frac{1}{3\times10^{-3}}\right)9.8 = h \times 1000 \times 9.8$$

After solving, we get

$$h = 4.76 \times 10^{-3} \text{ m.}$$

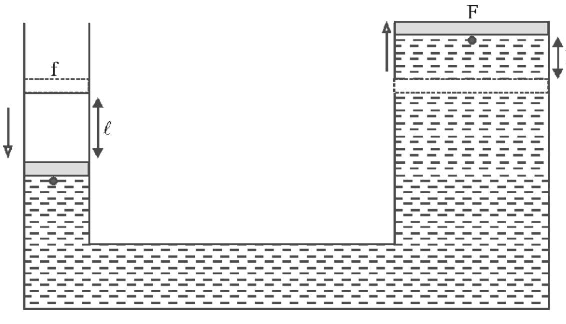
13. (7200) Radius of the pistons: $r = 1.25$ cm and R = 15 cm

As $\frac{f}{a} = \frac{F}{A}$

$\therefore$ $F = f\frac{A}{a} = f\frac{\pi R^2}{\pi r^2}$

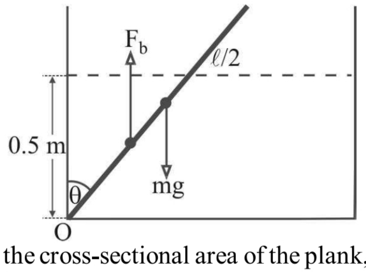
$= f\left(\frac{R}{r}\right)^2 = 50.0\left(\frac{15}{1.25}\right)^2$

$= 7200$ kg-wt



14. (45) Let y is the length of the plank inside water

$\therefore$ $y = \frac{0.5}{\cos\theta}$

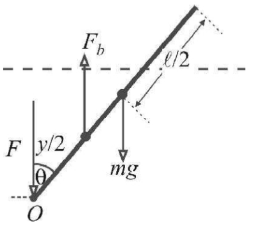


Let A be the cross-sectional area of the plank, then buoyant force on it

$F_b = V\rho_w g$
$= (Ay)\rho_w g$

Since plank is in rotational equilibrium, so

$\sum \vec{\tau}_o = 0$



or $mg \times \frac{\ell}{2}\sin\theta - F_b \times \frac{y}{2}\sin\theta = 0$

or $mg\ell - F_b \times y = 0$

$$(A\ell \times 0.5)g\ell - (Ay)\,d_w y = 0$$

or $$0.5\ell^2 = y^2$$

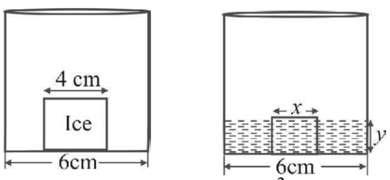
or $$0.5 \times (1)^2 = \left(\frac{0.5}{\cos\theta}\right)^2$$

$\Rightarrow$ $$\cos^2\theta = \frac{1}{2}$$

or $$\cos\theta = \frac{1}{\sqrt{2}}$$

or $$\theta = 45°.$$

15. (2.26)



Let size of ice block remaining is x^3 when it just about to float, then

$\therefore$ $$x^3 \times \rho_{ice}\, g = (x^2 y)\rho_w g \quad \ldots(i)$$

$\Rightarrow$ $$x^3 \rho_{ice} = x^2 y\, \rho_w$$

Also mass of ice melt = mass of water forms

$$(4^3 - x^3) \times \rho_{ice} = (\pi \times 3^2 \times y - x^2 y)\rho_w \quad \ldots(ii)$$

or $$4^3\rho_{ice} - x^3\rho_{ice} = \pi \times 3^2\, y\rho_w - x^2 y\rho_w$$

or $$4^3\rho_{ice} = \pi \times 3^2 y\rho_w$$

or $$y = \left(\frac{7.11\rho_{ice}}{\pi\rho_w}\right)$$

Substituting the value of y in equation (i), we get

$$x^3 \times \rho_{ice}\, g = x^2 \times \left(\frac{7.11\rho_{ice}}{\pi\rho_w}\right)\rho_w g$$

or $$x = 2.26 \text{ cm}.$$